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A Common Fixed Point in IFM-Space Using the Property CLR_g

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ABSTRACT: The purpose of this paper is to obtain a common fixed point theorem for weakly compatible self-map satisfying the property CLR_g using implicit relation on an Intuitionistic fuzzy metric space without involving the requirement of continuity of the maps which will also generalize and improve several results of metric spaces and fuzzy metric space into intuitionistic fuzzy metric spaces

KEYWORDS: Intuitionistic Fuzzy metric space, weakly compatible maps, property (CLR_g), implicit relations, common fixed point.

I. INTRODUCTION

The concept of fuzzy sets was introduced initially by Zadeh [11] in the year 1965. As a generalization of fuzzy sets, Atanassov [24] introduced the concept of Intuitionistic fuzzy sets. Further Coker [25] introduced the idea of the topology of Intuitionistic fuzzy sets. Intuitionistic fuzzy sets deals with both degree of nearness and degree of non-nearness. In 2004, Park [15] defined the notion of Intuitionistic fuzzy metric space with the help of continuous t-norms and continuous t-conorms as a generalization fuzzy metric space due to George and Veeramani [4]. Alea et al. defined the notion of Intuitionistic fuzzy metric space with the help of continuous t-norms and continuous t-conorms as a generalization of fuzzy metric space due to Kramosil and Michale [6]. In 2006, Turkoglu et al. [12] extended the notion of compatible mappings to IFM-spaces. In 2002, Aamri and El-Moutawakil [23] defined the notion of property (E.A) in metric spaces for self mappings which contained the class of non-compatible mappings in metric spaces. Most recently, Sintunavarat and Kumam [26] defined the notion of “Common limit in the range” property or CLR property in fuzzy metric spaces. Many authors have proved a number of fixed point theorems for different contractions in IFM-spaces. The aim of this paper is prove a common fixed point theorem in Intuitionistic fuzzy metric spaces using the notion of CLR_g property.

In this note, we prove common fixed point theorem for weakly compatible self –maps in Intuitionistic fuzzy metric spaces using the common limit in the range property of mappings called (CLR) property. Our results fuzzify, generalize and improve several results of metric spaces and fuzzy metric space into intuitionistic fuzzy metric spaces

II. PRELIMINARIES

Definition 1 [7] A binary operation $*$: $[0,1] \times [0,1] \rightarrow [0,1]$ is called a continuous t-norm if $([0, 1], *)$ is an abelian topological monoid with the unit 1 such that $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$ for all $a, b, c, d \in [0,1]$. Examples of t-norms are $a * b = ab$ and $a * b = \min \{a, b\}$

Definition 2 [7] A binary operation \diamond : $[0,1] \times [0,1] \rightarrow [0,1]$ is continuous t-conorm if \diamond is satisfying the following conditions:

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- 2.1 \diamond is commutative and associate
- 2.2 \diamond is continuous
- 2.3 $a \diamond 0 = a$ for all $a \in [0,1]$
- 2.4 $a \diamond b \leq c \diamond d$ Whenever $a \leq c$ and $b \leq d$ for all $a, b, c, d \in [0,1]$

Definition 3 [4] The 3-tuple $(X, M, *)$ is called a fuzzy metric space (FM-space) if X is an arbitrary set $*$ is a continuous t-norm and M is a fuzzy set in $X^2 \times (0, \infty)$ satisfying the following conditions for all $x, y, z \in X$ and $t, s > 0$.

- 3.1 $M(x, y, 0) > 0$
- 3.2 $M(x, y, t) = 1, \forall t > 0$ iff $x = y$
- 3.3 $M(x, y, t) = M(y, x, t)$,
- 3.4 $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$
- 3.5 $M(x, y, \cdot): [0, \infty] \rightarrow [0,1]$ is continuous.

Remark 1 since $*$ is continuous, it follows from (2.3.4) that the limit of a sequence in FM-space is uniquely determined

Definition 4 [16] A five –tuple $(X, M, N, *, \diamond)$ is said to be an Intuitionistic fuzzy metric space if X is an arbitrary set $*$ is a continuous t – norm, \diamond is a continuous t-conorm and M, N are fuzzy sets on $X^2 \times (0, \infty)$ satisfying the following conditions for all $x, y, z \in X, s, t > 0$

- 4.1 $M(x, y, t) + N(x, y, t) \leq 1$
- 4.2 $M(x, y, t) > 0$
- 4.3 $M(x, y, t) = M(y, x, t)$
- 4.4 $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$
- 4.5 $M(x, y, \cdot): (0, \infty) \rightarrow (0,1)$ is continuous
- 4.6 $N(x, y, t) > 0$
- 4.7 $N(x, y, t) = N(y, x, t)$
- 4.8 $N(x, y, t) \diamond N(y, z, s) \geq N(x, z, t + s)$
- 4.9 $N(x, y, \cdot): (0, \infty) \rightarrow (0,1]$ is continuous

Then (M, N) is called an Intuitionistic fuzzy metric on X , the function $M(x, y, t)$ and $N(x, y, t)$ denote the degree of nearness and the degree of non- nearness between x and y with respect to t respectively.

Remark 2 Every fuzzy metric space $(X, M, *)$ is an intuitionistic fuzzy metric space of the form $(X, M, 1 - M, *, \diamond)$ such that t-norm $*$ and t-conorm \diamond are associated i.e. $x \diamond y = 1 - ((1 - x) * (1 - y))$ for any $x, y \in [0,1]$ but the converse is not true.

Example 1 Induced Intuitionistic fuzzy metric space [1]

Let (X, d) be a metric space denote $a * b = ab$ and $a \diamond b = \min \{1, a + b\}$ for all $a, b \in [0,1]$ and let M , and N be fuzzy sets on $X^2 \times (0, \infty)$ defined as follows

$$M(x, y, t) = \frac{t}{t + d(x, y)}$$

$$N(x, y, t) = \frac{d(x, y)}{t + d(x, y)}$$

Then $(X, M, N, *, \diamond)$ is an Intuitionistic fuzzy metric space. We call this intuitionistic fuzzy metric induced by a metric d the standard intuitionistic fuzzy metric

Remark 3 Note that the above example holds even with the t-norm $a * b = \min \{a, b\}$ and the t-conorm $a \diamond b = \max \{a, b\}$

Example 2 Let $X = \mathbb{N}$ define $a * b = \max \{0, a + b - 1\}$ and $a \diamond b = a + b - ab$ for all $a, b \in [0,1]$ and let M and N be fuzzy sets on $X^2 \times (0, \infty)$ as follows

International Journal of Innovative Research in Science, Engineering and Technology

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Vol. 5, Issue 2, February 2016

$$M(x, y, t) = \begin{cases} \frac{x}{y} & \text{if } x \leq y \\ \frac{y}{x} & \text{if } y \leq x \end{cases}$$

$$N(x, y, t) = \begin{cases} \frac{y-x}{y} & \text{if } x \leq y \\ \frac{x-y}{x} & \text{if } y \leq x \end{cases}$$

for all $x, y \in X$ and $t > 0$ then $(X, M, N, *, \diamond)$ is an Intuitionistic fuzzy metric space.

Remark 4 Note that in above example, t -norm $*$ and t -conorm \diamond are not associated, and there exist no metric d on X satisfying $M(x, y, t) = \frac{t}{t+d(x,y)}$

$$\& N(x, y, t) = \frac{d(x,y)}{t+d(x,y)}$$

where $M(x, y, t)$ and $N(x, y, t)$ are defined in above example, also note that the above functions (M, N) is not an Intuitionistic metric with the t -norm and t -conorm defined as $a * b = \min \{a, b\}$ $a \diamond b = \max \{a, b\}$

Definition 5 [1] Let $(X, M, N, *, \diamond)$ be an intuitionistic fuzzy metric space then

(a) A sequence $\{x_n\}$ in X is said to be convergent to x in X if for each $\varepsilon > 0$ and each $t > 0$ there exists $n_0 \in \mathbb{N}$ such that

$$M(x_n, x, t) > 1 - \varepsilon$$

and

$$N(x_n, x, t) < \varepsilon \text{ For all } n \geq n_0$$

(b) An Intuitionistic fuzzy metric space in which every Cauchy sequence is convergent is said to be complete

Definition 6 [19] Let A and B maps from an Intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$ into itself. The maps A and B are said to be compatible (or asymptotically commuting) if for all $t > 0$

$$\lim_{n \rightarrow \infty} M(ABx_n, BAx_n, t) = 1$$

and

$$\lim_{n \rightarrow \infty} N(ABx_n, BAx_n, t) = 0$$

Whenever $\{x_n\}$ is a sequence in X such that $\lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Bx_n = z$ for some $z \in X$

Definition 7 [21] Two mappings A and B of an Intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$ into itself are R -weakly commuting provided there exists some real number R such that

$$M(ABx, BAx, t) \geq M(Ax, Bx, t/R)$$

and

$$N(ABx, BAx, t) \leq N(Ax, Bx, t/R) \text{ for each } x \in X \text{ and } t > 0$$

Definition 8 [20] Let f and g be two self mappings of an Intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$. We say that f and g satisfy the property (E.A) if there exists a sequence $\{x_n\}$ such that

$$\lim_{n \rightarrow \infty} fx_n = \lim_{n \rightarrow \infty} gx_n = t \text{ For some } t \in X$$

Definition 9 [20] Two self mappings f and g of an Intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$ are said to satisfy the property (CLR_g) if there exists a sequence $\{x_n\}$ in X such that

$$\lim_{n \rightarrow \infty} f(x_n) = \lim_{n \rightarrow \infty} g(x_n) = g(p) \text{ For some } p \in X$$

Similarly we can have the property (CLR_T) and the property (CLR_S) if mapping f and g is replaced by T and S respectively.

Lemma 1[1] In Intuitionistic fuzzy metric space X , $M(x, y, \cdot)$ is non decreasing and $N(x, y, \cdot)$ is non increasing for all $x, y \in X$

International Journal of Innovative Research in Science, Engineering and Technology

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Vol. 5, Issue 2, February 2016

Lemma 2 [1] Let $(X, M, N, *, \diamond)$ be an intuitionistic fuzzy metric space if there exist $k \in (0, 1)$ such that $M(x, y, kt) \geq M(x, y, t)$ and $N(x, y, kt) \leq N(x, y, t)$ for all $x, y \in X$, then $x = y$

Definition 10 We define an **implicit relation** as, let Φ_5 and Ψ_5 denote the set of all continuous functions from $[0, 1]^5 \rightarrow \mathbb{R}$ satisfying the conditions

Φ_1 : Φ is non-increasing in t_2, t_3, t_4 and t_5

Φ_2 : $\Phi(u, v, v, v, v) \geq 0 \Rightarrow u \geq v$ for $u, v \in [0, 1]$

and

Ψ_1 : Ψ is non-decreasing in t_2, t_3, t_4 and t_5

Ψ_2 : $\Psi(u, v, v, v, v) \geq 0 \Rightarrow u \leq v$ for $u, v \in [0, 1]$

Example $\Phi(t_1, t_2, t_3, t_4, t_5) = t_1 - \max\{t_2, t_3, t_4, t_5\}$
and $\Psi(t_1, t_2, t_3, t_4, t_5) = \min\{t_2, t_3, t_4, t_5, t_6\} - t_1$

III. MAIN RESULT

Theorem 1 Let A, B, S and T be self- maps of an Intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$ satisfying the following conditions:

(3.1) $A(X) \subseteq T(X), B(X) \subseteq S(X)$

(3.2) (A, S) and (B, T) are weakly compatible pairs

(3.3) (A, S) or (B, T) satisfy the property (CLR_s) and $(CLRT)$ respectively.

(3.4) For some $\Phi \in \Phi_5$ and $\Psi \in \Psi_5$, and for all $x, y \in X, t > 0$ & $k \in (0, 1)$,

$$\text{and } \Phi \left[\begin{array}{c} M(Ax, By, kt), M(Sx, Ty, t), \frac{M(By, Sx, t) + M(By, Ty, t)}{2} \\ \frac{M(Ax, Ty, t) + M(Sx, Ax, t)}{2}, M(Sx, Ax, t) \end{array} \right] \geq 0$$

$$\text{and } \Psi \left[\begin{array}{c} N(Ax, By, kt), N(Sx, Ty, t), \frac{N(By, Sx, t) + N(By, Ty, t)}{2} \\ \frac{N(Ax, Ty, t) + N(Sx, Ax, t)}{2}, N(Sx, Ax, t) \end{array} \right] \geq 0$$

If the range of one of the maps A, B, S or T is a complete subspace of X . then A, B, S and T have a unique common fixed point in X .

Proof: Without loss of generality, assume that $B(X) \subseteq S(X)$ and the pair (B, T) satisfies property (CLR_T) , then there exist a sequence $\{x_n\}$ in X such that Bx_n and Tx_n converges to Tx , for some x in X as $n \rightarrow \infty$. Since $B(X) \subseteq S(X)$, so there exist a sequence $\{y_n\}$ in X such that $Bx_n = Sy_n$, hence $Sy_n \rightarrow Tx$ as $n \rightarrow \infty$. We shall show that $\lim_{n \rightarrow \infty} Ay_n = Tx$.

Suppose $Ay_n \rightarrow w (\neq Tx)$ then by taking $x = y_n, y = x_n$ in (3.4)

$$\Phi \left[\begin{array}{c} M(Ay_n, Bx_n, kt), M(Sy_n, Tx_n, t), \frac{M(Bx_n, Sy_n, t) + M(Bx_n, Tx_n, t)}{2} \\ \frac{M(Ay_n, Tx_n, t) + M(Sy_n, Ay_n, t)}{2}, M(Sy_n, Ay_n, t) \end{array} \right] \geq 0$$

Letting $n \rightarrow \infty$ we get

$$\Phi \left[\begin{array}{c} M(w, Tx, kt), M(Tx, Tx, t), \frac{M(Tx, Tx, t) + M(Tx, Tx, t)}{2} \\ \frac{M(w, Tx, t) + M(Tx, w, t)}{2}, M(Tx, w, t) \end{array} \right] \geq 0$$

$$\Rightarrow \Phi[M(w, Tx, kt), 1, 1, M(w, Tx, t), M(w, Tx, t)] \geq 0$$

$$\Rightarrow \Phi[M(w, Tx, kt), M(w, Tx, t), M(w, Tx, t), M(w, Tx, t), M(w, Tx, t)] \geq 0 \text{ since } \Phi \text{ is non-increasing in } t_2 \text{ and } t_3$$

Therefore using the property Φ_2 , we have $M(w, Tx, kt) \geq M(w, Tx, t)$

Also,

$$\Psi \left[\begin{array}{c} N(Ay_n, Bx_n, kt), N(Sy_n, Tx_n, t), \frac{N(Bx_n, Sy_n, t) + N(Bx_n, Tx_n, t)}{2} \\ \frac{N(Ay_n, Tx_n, t) + N(Sy_n, Ay_n, t)}{2}, N(Sy_n, Ay_n, t) \end{array} \right] \geq 0$$

International Journal of Innovative Research in Science, Engineering and Technology

(A High Impact Factor, Monthly Peer Reviewed Journal)

Vol. 5, Issue 2, February 2016

Letting $n \rightarrow \infty$ we get

$$\Psi \left[\begin{array}{c} N(w, Tx, kt), N(Tx, Tx, t), \frac{N(Tx, Tx, t) + N(Tx, Tx, t)}{2} \\ \frac{N(Tx, Tx, t) + N(Tx, w, t)}{2}, N(Tx, w, t) \end{array} \right] \geq 0$$

$$\Rightarrow \Psi[N(w, Tx, kt), 0, 0, N(w, Tx, t), N(w, Tx, t)] \geq 0$$

$$\Rightarrow \Psi[N(w, Tx, kt), N(w, Tx, t), N(w, Tx, t), N(w, Tx, t), N(w, Tx, t)] \geq 0 \text{ since } \Psi \text{ is non-decreasing in } t_2 \text{ and } t_3$$

Therefore using the property Ψ_2 , we have $N(w, Tx, kt) \leq N(w, Tx, t)$

Thus by using lemma 2.2 we have $Tx = w$

This shows that $Ay_n \rightarrow Tx$ as $n \rightarrow \infty$.

Now suppose that $S(X)$ is a complete subspace of X then $Tx = Su$ for some $u \in X$. Subsequently we have,

$Ay_n \rightarrow Su, Bx_n \rightarrow Su, Tx_n \rightarrow Su$ and $Sy_n \rightarrow Su$ as $n \rightarrow \infty$

Again taking $x = u, y = x_n$ in (3.4), we have,

$$\Phi \left[\begin{array}{c} M(Au, Bx_n, kt), M(Su, Tx_n, t), \frac{M(Bx_n, Su, t) + M(Bx_n, Tx_n, t)}{2} \\ \frac{M(Au, Tx_n, t) + M(Su, Au, t)}{2}, M(Su, Au, t) \end{array} \right] \geq 0$$

Letting $n \rightarrow \infty$ we get

$$\Phi \left[\begin{array}{c} M(Au, Su, kt), M(Su, Su, t), \frac{M(Su, Su, t) + M(Su, Su, t)}{2} \\ \frac{M(Au, Su, t) + M(Su, Au, t)}{2}, M(Su, Au, t) \end{array} \right] \geq 0$$

$$\Rightarrow \Phi[M(Au, Su, kt), 1, 1, M(Au, Su, t), M(Au, Su, t)] \geq 0$$

$$\Rightarrow \Phi[M(Au, Su, kt), M(Au, Su, t), M(Au, Su, t), M(Au, Su, t), M(Au, Su, t)] \geq 0 \text{ since } \Phi \text{ is non-increasing in } t_2 \text{ and } t_3$$

Therefore using the property Φ_2 , we have $M(Au, Su, kt) \geq M(Au, Su, t)$

Also,

$$\Psi \left[\begin{array}{c} N(Au, Bx_n, kt), N(Su, Tx_n, t), \frac{N(Bx_n, Su, t) + N(Bx_n, Tx_n, t)}{2} \\ \frac{N(Au, Tx_n, t) + N(Su, Au, t)}{2}, N(Su, Au, t) \end{array} \right] \geq 0$$

Letting $n \rightarrow \infty$ we get

$$\Psi \left[\begin{array}{c} N(Au, Su, kt), N(Su, Su, t), \frac{N(Su, Su, t) + N(Su, Su, t)}{2} \\ \frac{N(Au, Su, t) + N(Su, Au, t)}{2}, N(Su, Au, t) \end{array} \right] \geq 0$$

$$\Rightarrow \Psi[M(Au, Su, kt), 0, 0, M(Au, Su, t), M(Au, Su, t)] \geq 0$$

$$\Rightarrow \Psi[M(Au, Su, kt), M(Au, Su, t), M(Au, Su, t), M(Au, Su, t), M(Au, Su, t)] \geq 0 \text{ since } \Psi \text{ is non-decreasing in } t_2 \text{ and } t_3$$

Therefore using the property Ψ_2 , we have $N(Au, Su, kt) \geq N(Au, Su, t)$

Therefore by using lemma 2.2 we have $Au = Su$

But weak compatibility of A and S implies $ASu = SAu$

this implies $AAu = ASu = SSu$

On the other hand since $A(X) \subseteq T(X)$, therefore there exist $u, v \in X$ such that $Au = Tv$. now we show that $Tv = Bv$.

Again taking $x = u, y = v$ (3.4) we have

$$\Phi \left[\begin{array}{c} M(Au, Bv, kt), M(Su, Tv, t), \frac{M(Bv, Su, t) + M(Bv, Tv, t)}{2} \\ \frac{M(Au, Tv, t) + M(Su, Au, t)}{2}, M(Su, Au, t) \end{array} \right] \geq 0$$

$$\Rightarrow \Phi \left[\begin{array}{c} M(Tv, Bv, kt), M(Tv, Tv, t), \frac{M(Bv, Tv, t) + M(Bv, Tv, t)}{2} \\ \frac{M(Tv, Tv, t) + M(Au, Au, t)}{2}, M(Au, Au, t) \end{array} \right] \geq 0$$

International Journal of Innovative Research in Science, Engineering and Technology

(A High Impact Factor, Monthly Peer Reviewed Journal)

Vol. 5, Issue 2, February 2016

$\Rightarrow \varphi[M(Tv, Bv, kt), 1, M(Tv, Bv, t), 1, 1] \geq 0$
 $\Rightarrow \varphi[M(Tv, Bv, kt), M(Tv, Bv, t), M(Tv, Bv, t), M(Tv, Bv, t), M(Tv, Bv, t)] \geq 0$ since Φ is non-increasing in t_2, t_4 and t_5

Therefore using the property φ_2 , we have $M(Tv, Bv, kt) \geq M(Tv, Bv, t)$

Also,

$$\Psi \left[\begin{array}{c} N(Au, Bv, kt), N(Su, Tv, t), \frac{N(Bv, Su, t) + N(Bv, Tv, t)}{2} \\ \frac{N(Au, Tv, t) + N(Su, Au, t)}{2}, N(Su, Au, t) \end{array} \right] \geq 0$$

$$\Rightarrow \Psi \left[\begin{array}{c} N(Tv, Bv, kt), N(Tv, Tv, t), \frac{N(Bv, Tv, t) + N(Bv, Tv, t)}{2} \\ \frac{N(Tv, Tv, t) + N(Au, Au, t)}{2}, N(Au, Au, t) \end{array} \right] \geq 0$$

$\Rightarrow \Psi[N(Tv, Bv, kt), 0, N(Tv, Bv, t), 0, 0] \geq 0$

$\Rightarrow \Psi[M(Tv, Bv, kt), M(Tv, Bv, t), M(Tv, Bv, t), M(Tv, Bv, t), M(Tv, Bv, t)] \geq 0$ since Ψ is non-decreasing in t_2, t_4 and t_5

Therefore using the property Ψ_2 , We have $N(Tv, Bv, kt) \leq N(Tv, Bv, t)$

Therefore by using lemma 2.2 we have $Tv = Bv$.

This implies that $Au = Su = Tv = Bv$

But the weak compatibility of B and T implies that $BTv = TBv$ which implies $TBv = BTv = BBv$.

Now let us show that Au is a common fixed point of A, B, S and T.

Again in (3.4) taking $x = Au$, $y = u$ it follows

$$\Phi \left[\begin{array}{c} M(AAu, Au, kt), M(AAu, Au, t), \frac{M(Au, AAu, t) + M(Au, Au, t)}{2} \\ \frac{M(AAu, Au, t) + M(AAu, Au, t)}{2}, M(AAu, AAu, t) \end{array} \right] \geq 0$$

$$\Rightarrow \Phi \left[\begin{array}{c} M(AAu, Au, kt), M(AAu, Au, t), \frac{M(Au, AAu, t) + 1}{2} \\ \frac{M(AAu, Au, t) + 1}{2}, 1 \end{array} \right] \geq 0$$

$$\Rightarrow \Phi \left[\begin{array}{c} M(AAu, Au, kt), M(AAu, Au, t), M(AAu, Au, t) \\ M(AAu, Au, t), 1 \end{array} \right] \geq 0$$

$$\Rightarrow \Phi \left[\begin{array}{c} M(AAu, Au, kt), M(AAu, Au, t), M(AAu, Au, t) \\ M(AAu, Au, t), M(AAu, Au, t) \end{array} \right] \geq 0$$
 since Φ is non-increasing in t_5 .

Therefore using the property φ_2 , we have $M(AAu, Au, kt) \geq M(AAu, Au, t)$

Also,

$$\Psi \left[\begin{array}{c} N(AAu, Au, kt), N(AAu, Au, t), \frac{N(Au, AAu, t) + N(Au, Au, t)}{2} \\ \frac{N(AAu, Au, t) + N(AAu, Au, t)}{2}, N(AAu, AAu, t) \end{array} \right] \geq 0$$

$$\Rightarrow \Psi \left[\begin{array}{c} N(AAu, Au, kt), N(AAu, Au, t), \frac{N(Au, AAu, t) + 0}{2} \\ \frac{N(AAu, Au, t) + 0}{2}, 0 \end{array} \right] \geq 0$$

$$\Rightarrow \Psi \left[\begin{array}{c} N(AAu, Au, kt), N(AAu, Au, t), N(AAu, Au, t) \\ N(AAu, Au, t), 0 \end{array} \right] \geq 0$$

$$\Rightarrow \Psi \left[\begin{array}{c} N(AAu, Au, kt), N(AAu, Au, t), N(AAu, Au, t) \\ N(AAu, Au, t), N(AAu, Au, t) \end{array} \right] \geq 0$$
 since Ψ is non-increasing in t_5 .

Therefore using the property Ψ_2 , We have $N(AAu, Au, kt) \leq N(AAu, Au, t)$.

Therefore by using lemma 2.2 we have $AAu = Au$.

Hence $Au = AAu = SAu$ and Au is a common fixed point of A and S.

Similarly we can prove that Bv is a common fixed point of B and T.

Now since $Au = Bv$, we conclude that Au is a common fixed point of A, B, S and T.

International Journal of Innovative Research in Science, Engineering and Technology

(A High Impact Factor, Monthly Peer Reviewed Journal)

Vol. 5, Issue 2, February 2016

The proof is similar when $T(X)$ is assumed to be complete subspace of X . The cases in which $A(X)$ or $B(X)$ is complete subspace of X are similar to the cases in which $T(X)$ or $S(X)$ respectively is complete since $A(X) \subseteq T(X)$ and $B(X) \subseteq S(X)$.

For uniqueness of fixed point, let $Au = Bu = Tu = Su = w_1$ and $Av = Bv = Tv = Sv = w_2$

Then (3.4) gives

$$\begin{aligned} & \Phi \left[\begin{array}{c} M(Au, Bv, kt), M(Su, Tv, t), \frac{M(Bv, Su, t) + M(Bv, Tv, t)}{2}, \\ \frac{M(Au, Tv, t) + M(Su, Au, t)}{2}, M(Su, Au, t) \end{array} \right] \geq 0 \\ \Rightarrow & \Phi \left[\begin{array}{c} M(w_1, w_2, kt), M(w_1, w_2, t), \frac{M(w_2, w_1, t) + M(w_2, w_2, t)}{2}, \\ \frac{M(w_1, w_2, t) + M(w_1, w_1, t)}{2}, M(w_1, w_1, t) \end{array} \right] \geq 0 \\ \Rightarrow & \Phi \left[\begin{array}{c} M(w_1, w_2, kt), M(w_1, w_2, t), \frac{M(w_2, w_1, t) + 1}{2}, \\ \frac{M(w_1, w_2, t) + 1}{2}, 1 \end{array} \right] \geq 0 \\ \Rightarrow & \Phi \left[\begin{array}{c} M(w_1, w_2, kt), M(w_1, w_2, t), M(w_1, w_2, t), \\ M(w_1, w_2, t), 1 \end{array} \right] \geq 0 \\ \Rightarrow & \Phi \left[\begin{array}{c} M(w_1, w_2, kt), M(w_1, w_2, t), M(w_1, w_2, t), \\ M(w_1, w_2, t), M(w_1, w_2, t) \end{array} \right] \geq 0 \text{ since } \Phi \text{ is non-increasing in } t_5. \end{aligned}$$

Therefore using the property ϕ_2 , we have $M(w_1, w_2, kt) \geq M(w_1, w_2, t)$.

Also,

$$\begin{aligned} & \Psi \left[\begin{array}{c} N(Au, Bv, kt), N(Su, Tv, t), \frac{N(Bv, Su, t) + N(Bv, Tv, t)}{2}, \\ \frac{N(Au, Tv, t) + N(Su, Au, t)}{2}, N(Su, Au, t) \end{array} \right] \geq 0 \\ \Rightarrow & \Psi \left[\begin{array}{c} N(w_1, w_2, kt), N(w_1, w_2, t), \frac{N(w_2, w_1, t) + N(w_2, w_2, t)}{2}, \\ \frac{N(w_1, w_2, t) + N(w_1, w_1, t)}{2}, N(w_1, w_1, t) \end{array} \right] \geq 0 \\ \Rightarrow & \Psi \left[\begin{array}{c} N(w_1, w_2, kt), N(w_1, w_2, t), \frac{N(w_2, w_1, t) + 0}{2}, \\ \frac{N(w_1, w_2, t) + 0}{2}, 0 \end{array} \right] \geq 0 \\ \Rightarrow & \Psi \left[\begin{array}{c} N(w_1, w_2, kt), N(w_1, w_2, t), N(w_1, w_2, t), \\ N(w_1, w_2, t), 0 \end{array} \right] \geq 0 \\ \Rightarrow & \Psi \left[\begin{array}{c} N(w_1, w_2, kt), N(w_1, w_2, t), N(w_1, w_2, t), \\ N(w_1, w_2, t), N(w_1, w_2, t) \end{array} \right] \geq 0 \text{ since } \Psi \text{ is non-decreasing in } t_5. \end{aligned}$$

Therefore using the property Ψ_2 ,

we have $N(w_1, w_2, kt) \leq N(w_1, w_2, t)$.

And by using lemma 2.2 we have $w_1 = w_2$.

Hence the common fixed point is unique.

IV. CONCLUSION

The result on a common fixed point is successfully obtained for weakly compatible maps in Intuitionistic fuzzy metric spaces using the common limit in the range property of mappings called (CLR) property which also generalize and improve several results of metric spaces and fuzzy metric space into intuitionistic fuzzy metric spaces.

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